In Situ Soil Response to Vibratory Loading and Its Relationship to Roller-Measured Soil Stiffness

Michael A. Mooney and Robert V. Rinehart

Abstract: An investigation was conducted to characterize and relate in situ soil stress-strain behavior to roller-measured soil stiffness. Continuous assessment of soil stiffness via roller vibration monitoring has the potential to significantly advance performance based quality assurance of earthwork. One vertically homogeneous and two layered test beds were carefully constructed with embedded sensors for the field testing program. Total normal stress and strain measurements at multiple depths reveal complex triaxial soil behavior during vibratory roller loading. Measured cyclic strain amplitudes were 15–25% of those measured during static roller passes due to viscoelasticity and curved drum/soil interaction. Low amplitude vibratory roller loading induces nonlinear in situ modulus behavior. Roller-measured stiffness and its dependence on excitation force is influenced by the stress-dependent modulus function of each soil, the varying drum/soil contact area, and by layer characteristics (modulus ratio, thickness) when layering is present. On vertically homogeneous clayey sand, roller-measured stiffness decreased with increasing excitation force, a behavior attributed to stress-dependent modulus reduction observed in situ. On the crushed rock over silt test bed, roller-measured stiffness increased with increasing excitation force despite the mild stress-dependent modulus reduction observed in the crushed rock. In this case, the stiffer crushed rock takes on a greater portion of the load, resulting in the increase in roller-measured stiffness.

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CE Database subject headings: In situ tests; Vibration; Soil properties; Stiffness; Stress strain relations.

Introduction

Monitoring of roller vibration has been used for more than 25 years during soil compaction operations to provide what is termed continuous compaction control (CCC). Combined with positioning data [via wheel encoders or global positioning system (GPS)] and on-board PCs with geographic information system (GIS) capability, roller-based measurement of soil properties provides real time quality control (QC) for the contractor and usable information for quality assurance (QA). In the early years of CCC, roller-based measures of soil compaction were heuristically determined relative indices, e.g., the compaction meter value (CMV) based on the concept that the harmonic content of the drum vibration signal increased with compaction (Forssblad 1980). More recently, roller vibration data has been used to extract continuous measurement of soil stiffness (Kröber et al. 2001; Anderegg and Kaufmann 2004; Mooney and Rinehart 2007). With measured stiffness comes some implication and potential that roller-measured soil properties are related to pavement performance parameters, e.g., design moduli. If so, continuous assessment of soil stiffness would signal a significant leap forward in QA, particularly in performance based QA and closer coupling of design with construction.

The relationship between roller-measured stiffness and in situ soil behavior is complex. Roller/soil interaction is highly nonlinear, involves multidegree-of-freedom kinematics, transient response, and decoupling between the drum and soil (Adam 1996; Anderegg 1997; van Susante and Mooney 2008). Roller-measured stiffness provides a composite reflection of the soil to a depth that is greater than typical 15–30 cm lift thickness, and often conveys the composite response of layered strata, e.g., base over subbase or subgrade. Further, roller-measured soil stiffness has been found to vary with excitation force amplitude and frequency (Adam 1996; Anderegg and Kaufmann 2004; Kopf and Erdmann 2005; Mooney and Rinehart 2007). This observed behavior is particularly pertinent because of the recent introduction of intelligent compaction (IC) where the excitation force amplitude and frequency are continuously varied in an attempt to promote faster and more uniform compaction. To advance the understanding and capabilities of CCC in vertically homogeneous and layered earth structures as well as the measurement of soil properties during IC, the relationship between roller-measured soil stiffness and in situ stress-strain response must be explored and characterized.

This paper presents the results of an investigation to characterize vibratory roller induced stress-strain response in compacted soil and to relate in situ stress-strain response to roller based stiffness measurements. The focus of this study is on the assessment of fully compacted soil, and not on the compaction process itself. The intent is to pursue roller-measured soil stiffness determined in a proof roll capacity to predict performance of the earthwork within the pavement system, i.e., performance-based assessment. Stress and strain sensors were placed at multiple depths within both vertically homogeneous and heterogeneous (layered) test beds. Triaxial stress-strain behavior was captured during static and vibratory passes of two commercially available IC rollers over a range of typical excitation force frequencies and amplitudes. The complex nature of triaxial stress-strain generation...
Table 1. Summary of Test Beds

<table>
<thead>
<tr>
<th>Test bed geometry</th>
<th>Clayey sand</th>
<th>Gravelly sand over clay</th>
<th>Crushed rock over silt</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length (m)</td>
<td>30</td>
<td>25</td>
<td>35</td>
</tr>
<tr>
<td>Width (m)</td>
<td>3.5</td>
<td>5</td>
<td>3.5</td>
</tr>
<tr>
<td>Depth (m)</td>
<td>1.0</td>
<td>0.5</td>
<td>0.72</td>
</tr>
<tr>
<td>Layer thickness (m)</td>
<td>0.2–0.3</td>
<td>0.25</td>
<td>0.2–0.3</td>
</tr>
<tr>
<td>Number of layers</td>
<td>4</td>
<td>2</td>
<td>3^b</td>
</tr>
</tbody>
</table>

Upper material soil characteristics

<table>
<thead>
<tr>
<th>Soil class, (USCS)</th>
<th>SC</th>
<th>SP-SM</th>
<th>SM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Percent passing 200 (%)</td>
<td>32</td>
<td>10</td>
<td>9</td>
</tr>
<tr>
<td>$C_{v}$, $C_{c}$</td>
<td>—</td>
<td>22, 0.7</td>
<td>66, 4</td>
</tr>
<tr>
<td>LL, PI</td>
<td>32, 14</td>
<td>NP</td>
<td>NP</td>
</tr>
</tbody>
</table>

Lower material soil characteristics

<table>
<thead>
<tr>
<th>Soil class, (USCS)</th>
<th>—</th>
<th>CL</th>
<th>ML</th>
</tr>
</thead>
<tbody>
<tr>
<td>Percent passing 200 (%)</td>
<td>—</td>
<td>56</td>
<td>52</td>
</tr>
<tr>
<td>$C_{v}$, $C_{c}$</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>LL, PI</td>
<td>—</td>
<td>26, 10</td>
<td>31, 1</td>
</tr>
</tbody>
</table>

^a Of top material; lower material extends beyond measurement depth of roller.

^b Additional layers up to total thickness of 1.5 m were placed.

as a roller passes is characterized. A deformation modulus from in situ stress-strain data is extracted and compared to roller-measured stiffness over a range of operating excitation force amplitudes and frequencies to provide insight into roller/soil interaction and the nature of roller-measured stiffness.

Test Program

Three test beds were carefully constructed with embedded stress and strain sensors to explore in situ stress-strain behavior and its relationship to roller-measured soil stiffness. The soil properties and test bed dimensions are summarized in Table 1; sensor locations are depicted in Fig. 1. A clayey sand test bed was constructed to represent a typical embankment earth structure with vertical homogeneity. The gravely sand over clay and crushed rock over silt test beds were constructed to represent the typical base/subbase over subgrade layered systems encountered in practice.

Earth pressure cells (EPCs) and modified LVDTs were installed during test bed construction at the depths shown in Fig. 1 to measure total normal stress and normal strain. Both types of sensors are capable of capturing the static (due to roller weight) and cyclic (due to drum vibration) response of the soil. Normal stress and strain were measured in three orthogonal directions, and are denoted by $\sigma_z$ and $\varepsilon_i$ ($i=x, y, \text{and } z$). Here, $z$ is considered positive downward from the soil surface, $x$ is positive in the direction of roller travel, and $y$ is positive into the paper. The construction of the test beds, installation of sensors, and verification of sensor accuracy and efficacy are described in detail elsewhere (Rinehart and Mooney 2009, Mooney and Miller 2008). Roller-measured stiffness and in situ stress-strain data were simultaneously collected during numerous roller passes over fully compacted test bed layers.

Roller Characteristics and Stiffness Measurement

Ammann and Bomag IC rollers were used in this study; the applicable roller parameters are summarized in Table 2. Vibratory excitation is created by a rotating eccentric mass configuration within the drum. The vertical component of the centrifugal force, $F(t)$, is given by Eq. (1) where $F_{ce}=amplitude$ of $F(t)$; $\Omega = circular excitation frequency (rad/s)$; $m_{ce}=eccentric mass moment$; and $t=time$. More details regarding eccentric excitation are available in the literature (Adam and Kopf 2004; Mooney and Adam 2007).

\[
F(t) = F_{ce} \cos(\Omega t) = m_{ce} \Omega^2 \cos(\Omega t) \tag{1}
\]

The measurement of soil stiffness by the Ammann and Bomag rollers is briefly described here; the reader is referred elsewhere for detailed descriptions (Krober et al. 2001; Anderegg and Kaufmann 2004; Mooney and Adam 2007). Roller-measured soil stiffness is derived from instrumentation of the vibrating drum (i.e., accelerometers, rotary encoders, see Rinehart and Mooney 2008) coupled with lumped parameter modeling, and is extracted from the vertical response of the drum as shown in Fig. 2(a). Per equi-

Table 2. Summary of Roller Parameters

<table>
<thead>
<tr>
<th>Roller</th>
<th>Ammann</th>
<th>Bomag</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drum length (m)</td>
<td>2.20</td>
<td>2.13</td>
</tr>
<tr>
<td>Drum radius (m)</td>
<td>0.75</td>
<td>0.75</td>
</tr>
<tr>
<td>Static mass (kg)</td>
<td>11,500</td>
<td>14,900</td>
</tr>
<tr>
<td>Static linear load (kN/m) under drum</td>
<td>31.5</td>
<td>42.4</td>
</tr>
<tr>
<td>Operating frequency (Hz)</td>
<td>20–34</td>
<td>28</td>
</tr>
<tr>
<td>$F_{ce}$ (kN)</td>
<td>0–277a</td>
<td>0–365b</td>
</tr>
</tbody>
</table>

^a 100 constant amplitude increments.

^b Six constant amplitude increments.

Fig. 1. Schematic of test beds constructed with embedded instrumentation: (a) clayey sand; (b) gravely sand over clay test bed; and (c) crushed rock over silt test bed
librium, and making no assumptions about the soil, the contact force (i.e., transmitted to the soil) is determined according to Eq. (2)

\[ F_y = F_{ev} \cos(\Omega t) + (m_f + m_d)g - m_d \ddot{z}_d - m_f \ddot{z}_f \]  \hspace{1cm} (2)

where \( m_f \) and \( m_d \)=mass of the frame and drum, respectively; \( g \)=acceleration due to gravity; \( \dot{m}_f \) and \( \dot{m}_d \)=accelerations of the frame and drum, respectively. Frame inertia \( (m_f \ddot{z}_f) \) has been shown to be small and is often neglected. Similarly, dynamic forces from the drum-frame suspension are small and neglected. By monitoring the position of the rotating eccentric mass and measuring the drum acceleration, it is possible to produce the force-deflection \( F_{vib} \) behavior of the drum in real time. Fig. 2(b) shows typical \( F_{vib} \) paths for fully coupled drum/soil (top) and partially uncoupled or loss of contact (bottom) operational modes. Loss of contact behavior, where the drum periodically lifts off the soil (once per eccentric revolution), is common at higher \( F_{vib} \) levels or when operating on stiff soils. The soil stiffness parameter \( E_{vib} \) developed by Bomag is based on a tangent stiffness from the loading portion of the curve \([k_1 \text{ in Fig. 2(b)}].\)

The reported Bomag \( E_{vib} \) parameter is the modulus equivalent of an elastic half-space that exhibits a stiffness \( k_1 \) determined using Lundberg’s (1939) cylinder/elastic half space solution (Kröber et al. 2001). The soil stiffness parameter \( k_2 \) developed by Ammann [shown as \( k_2 \) in Fig. 2(b)] is a secant stiffness from the point of zero deflection (under static loading) and static force through the point of maximum deflection (Anderegg and Kaufmann 2004). Both \( E_{vib} \) and \( k_2 \) are computed in real time onboard the roller and form the basis of roller-integrated CCC.

**Roller-Induced Stress-Strain Field Results**

A vibratory roller operating at typical speeds (0.5–1.0 m/s), excitation frequencies \((f=25–35 \text{ Hz})\), and “low” excitation force magnitudes \((F_{ev}=30–90 \text{ kN})\) induces a complex triaxial stress-strain condition in the soil. Fig. 3 presents triaxial total normal stress \( \sigma_i \) and normal strain \( \varepsilon_i \), data measured at a depth of \( z =0.13 \text{ m} \) in the gravelly sand test bed during both static and vibratory Ammann roller passes \((f=30 \text{ Hz}, F_{ev}=67 \text{ kN})\). All reported \( \sigma_i \) and \( \varepsilon_i \) are measured over a 100 mm earth pressure cell diameter or strain gauge length, and therefore reflect average values. The difference between these average values and values over infinitesimal dimensions has been shown to be minimal for these loading geometries (Mooney and Miller 2008) and does not affect the interpretation of the results.

It should also be noted that total normal stresses were measured. These soils are partially saturated and suction induced pore pressures are likely present. The measurement of pore water and air pressures in partially saturated soils under static and dynamic loading is extremely challenging and not yet reliably possible in the field. Nevertheless, there is considerable value in characterizing and evaluating total normal stresses. In addition, shear stresses and strains are induced by the roller but cannot be directly measured in the field. However, the results and analysis

![Fig. 2. (a) Forces acting on drum during vibration; (b) drum/soil contact force versus drum displacement response for contact (top) and partial loss of contact modes (bottom).](image)

![Fig. 3. Total normal stresses and strains measured during Ammann static and vibratory \((f=30 \text{ Hz}, F_{ev}=33 \text{ kN})\) passes on gravelly sand over clay test bed.](image)
The observed offset in $\sigma_z$ is attributed to residual or “locked in” stresses evident in the static pass but released during vibration. Compaction-induced residual stresses have been shown by D’Appolonia et al. (1969) and Duncan et al. (1991). The near surface release of locked in stress and/or loosening of soil is commonly observed on compacted soils, and is the reason why low amplitude vibration and static roller passes are recommended toward the end of compaction.

A significant finding in this study was the low magnitude of cyclic strains $\varepsilon_{cyc}$ induced by drum vibration compared to strains induced by static drum loading. Cyclic stress $\sigma_{cyc}$ magnitudes were found to be on the same order as those induced by static loading, and are consistent with the relative magnitudes of $F_{stat}$ versus static weight. $\varepsilon_{cyc}$ levels, however, were only 15–25% of those induced by static loading. This is important because roller based stiffness measures are based on cyclic drum deformation. Though the total stress and strain state in the soil is important for general understanding of soil behavior (i.e., stress-strain dependent response), the cyclic portion of strain is a reflection of the drum deformation used in the roller-based stiffness calculation.

The small magnitudes of $\varepsilon_{cyc}$ (relative to $\varepsilon_s$) can be attributed to curved drum interaction with the soil and to viscoelasticity. During vibration, the drum/soil contact force equals the static force $F_s$ plus a cyclic force that is a function of $F_{stat}$ and drum inertia (Mooney and Rinehart 2007). Contact stress and contact area are nonlinearly related to force and vertical drum deflection, and lead to a hardening type relationship between applied force and drum deflection as depicted in Fig. 4. The deflections (and in turn strains) due to drum vibration are smaller than the deflections resulting from static loading. The $\varepsilon_s$ response is also consistent with classical viscoelastic behavior of soil when subjected to cyclic loading (Ishihara 1996; Brown 1996). Viscoelasticity has been extensively used to model foundation/soil (Gazetas 1983; Wolf 1994) and drum/soil interaction (Yoo and Selig 1979; Adam 1996; Anderegg 1997; Mooney and Rinehart 2007). The time dependent nature of viscoelastic soil response significantly attenuates strain during 30 Hz cyclic loading compared to the ~1 Hz static loading passes.

Two additional behavioral characteristics are worth noting. The soil immediately in front of the drum experiences vertical ($z$) extension and longitudinal ($x$) compression. This bow wave phenomenon is due to the traveling nature of the roller and was observed in measured $\varepsilon_z$ and $\varepsilon_x$. The bow wave leads to $x$ direction asymmetry in the strain field influenced by the roller. In addition, the stress-strain data in Fig. 3 recorded beneath the center of the 2.1 m long drum suggests that the soil approaches plane strain conditions. Eccentric excitation loads the soil in the $x$ and $z$ directions only. Both total and cyclic levels of $\sigma_x$ ($10^{-3}$ and $10^{-4}$, respectively) were at least an order of magnitude lower than total and cyclic $\sigma_z$ and $\varepsilon_z$ levels ($10^{-2}$ and $10^{-3}$, respectively). These levels coupled with the confinement-induced high values of $\sigma_y$ (relative to $\sigma_x$, $\sigma_z$) suggest near plane strain conditions exist.

This plane strain condition does not exist beneath the drum edges, meaning the stress state varies along the drum length (Rinehart et al. 2008).

The stress-strain field during static and vibratory passes over compacted clayey sand is shown in Fig. 5. The measured triaxial stress and strain at a depth of 0.14 m in the clayey test bed were induced by an Ammann roller pass ($f=30$ Hz, $F_{ev}=87$ kN). The more fine-grained material in the sandy clay test bed (32% fines) exhibited a softer response than the gravelly sand test bed (30% gravel, 10% fines). Peak $\sigma_z$ values were less in the clayey sand than in the gravelly sand (Fig. 3) despite $F_{ev}$ being 30% greater.

Measured $\varepsilon_z$ and $\varepsilon_x$ during vibratory loading were noticeably greater than those measured during static loading (previous pass). This behavior was consistently observed on the clayey sand and not observed on the gravelly sand and crushed rock. This vibration-induced softening response is also characterized as elastic in that pre- and postpass strain levels were similar. Similar behavior was observed by Brandl et al. (2005) based on deflection measurements in a cohesive soil. A plausible explanation for this behavior is modulus degradation due to the generation of pore air and/or pore water pressure during vibration. While instrumentation to measure pore pressures was not employed in this study and therefore cannot be confirmed, both pore air and water pressures are known to occur in partially saturated soils with appreciable fines (Unno et al. 2006).

Modulus reduction leads to increased drum penetration into the soil and a consequential increase in contact area. As a result, mean $\sigma_z$ levels during vibratory loading were greater than during static loading on the clayey sand. This is explained by considering the nonlinear $\sigma_z$ profile predicted by the Hertzian solution for a cylinder in contact with a homogeneous, isotropic, linear elastic half space (Johnson 1987). The normal stresses for $z=0, x=0$ are given by

$$\sigma_z = \frac{p_0 a}{z} \left( \frac{a^2 + 2z^2}{\sqrt{a^2 + z^2}} - 2z \right)$$

(3)

$$\sigma_x = \frac{p_0}{\sqrt{a^2 + z^2}}$$

(4)

$$\sigma_z = \nu (\sigma_x + \sigma_z)$$

(5)
where \( p_{z_{a}} \) = maximum normal contact stress at the surface; \( P \) = load per unit drum length applied at the cylinder/soil interface; \( a \) = half contact width; and \( z \) = depth below the surface. Based on the author’s experience and from data in Brandl et al. (2005), a reasonable approximation of \( z/a \) for the static and vibratory roller passes in Fig. 5 are 1.5 and 1.1, respectively, with the latter varying between 1.05 and 1.15 during a cycle of vibration. As shown by the normalized \( \sigma_z \) response in Fig. 5, the decrease in \( z/a \) results in an increase in \( \sigma_z \). A similar argument applies to \( \sigma_y \) (not shown here).

**Vertical Deformation Modulus**

Roller-based stiffness is derived from cyclic surface deformation \( z_d \) (Fig. 2), and is indirectly influenced by the soil response in the \( x \) and \( y \) directions. Similarly, a vertical dynamic deformation modulus \( M \) can be extracted from the in situ \( \sigma_z-\varepsilon_z \) response to vibratory loading. In the present work, the extracted \( M \) values are not constitutive (e.g., Young’s modulus); rather, \( M \) is akin to a partially constrained dynamic modulus that is influenced by the \( \sigma_y-\varepsilon_y \) and \( \sigma_z-\varepsilon_z \) fields. The extraction of constitutive soil parameters accounting for inertia, viscoelasticity, plasticity, suction and pore pressures, and stress path is complex and the topic of ongoing research. Fig. 6 illustrates the measures of \( M \) evaluated in this study as determined from the \( \sigma_z-\varepsilon_z \) response during Ammann vibratory roller passes over clayey sand \((z=0.14 \text{ m}, f=30 \text{ Hz}, F_{ev}=87 \text{ kN})\) and gravelly sand \((z=0.13 \text{ m}, f=30 \text{ Hz}, F_{ev}=33 \text{ kN})\) test beds. \( M_T \) is the tangent modulus representative of the loading portion of the \( \sigma_z-\varepsilon_z \) response. As defined, \( M_T \) is not strictly unique but was chosen to be comparable to Bomag \( E_{vb} \) (see Figs. 2 and 6). Experience has shown that the loading portion of the

**Fig. 5.** Total normal stresses and strains measured during Ammann static and vibratory \((f=30 \text{ Hz}, F_{ev}=87 \text{ kN})\) passes on clayey sand test bed

**Fig. 6.** Vertical cyclic stress-strain response and deformation moduli \( M \) from Ammann roller pass over: (a) clayey sand test bed \((z=0.14 \text{ m}, f=30 \text{ Hz}, F_{ev}=87 \text{ kN})\); (b) gravelly sand test bed \((z=0.13 \text{ m}, f=30 \text{ Hz}, F_{ev}=33 \text{ kN})\)
\[ \sigma_{zc} - e_{zc}, \] is quite linear, reducing the uncertainty due to this definition. \( M_S \) is the secant modulus from zero \( \sigma_{zc} - e_{zc} \) through the point of maximum \( e_{zc} \) and is comparable to Ammann \( k_s \).

The presentation of in situ \( M_T \) and \( M_S \) values during Ammann and Bomag roller passes reveals important insights about soil behavior during vibratory loading and helps to explain the nature of roller-based stiffness. Fig. 7 presents \( M_T \) and \( M_S \) values determined at three depths during passes of the Ammann [Figs. 7(a and b)] and Bomag [Figs. 7(c and d)] rollers over the clayey sand. The \( M \) versus \( F_{ev} \) data for the Ammann passes was characterized statistically with power relationships (i.e., \( M_T = b(F_{ev})^c \)) at each depth, while the Bomag pass data was characterized with linear relationships (i.e., \( M_T = a+b(F_{ev}) \)). Values of \( R^2 \) are provided in Fig. 7. Note that the Bomag roller is capable of greater \( F_{ev} \) than the Ammann roller used here, and upon inspection of Fig. 7, the \( M \) versus \( F_{ev} \) relationship from both rollers appears linear for \( F_{ev} > 100 \) kN.

\( M_T \) and \( M_S \) generally increase with depth and decrease with increasing \( F_{ev} \). For both rollers, \( M_T \) varies by a factor of two for \( F_{ev} \) values typically employed in the field and by a factor of two over the depths measured (0.14–0.65 m). The sensitivity of \( M_S \) values to depth and to \( F_{ev} \) was similarly small to \( M_T \) with the exception that the difference in Bomag induced \( M_S \) with depth increased with \( F_{ev} \). The observed \( M_T \) and \( M_S \) behavior with \( F_{ev} \) are a field manifestation of the stress dependence of soil modulus that has been well established in the laboratory (Ishihara 1996; Andrei et al. 2004). For finer grained granular and cohesive soils, the decrease in modulus with increasing shear stress typically outweighs the increase in modulus due to increasing effective confining stress (Andrei et al. 2004). During vibratory loading here, the substantial levels of \( \sigma_e \) and \( \sigma_c \) coupled with very low values of \( \sigma_s \) at \( x = 0 \) (see Fig. 5) lead to significant levels of deviatoric stress that appears to cause the decrease in \( M \) with increasing \( F_{ev} \). The results here and below for other test beds indicate that roller measured stiffness—even at low \( F_{ev} \)—reflects significant nonlinear modulus with depth. If roller-measured stiffness values are to be used to analyze pavement performance, the differences between roller-induced and traffic-induced stress conditions and their effect on modulus must be considered.

**Relationship to Roller-Based Stiffness**

One main impetus for this study was to relate roller-measured stiffness to in situ stress-strain response and deformation modulus \( M \). Previous research has shown that roller-based measures of soil stiffness vary with \( F_{ev} \) (Kopf and Erdmann 2005; Adam and Kopf 2004; Mooney and Rinehart 2007). This is also illustrated in Fig. 8 where Ammann \( k_s \) decreases with increasing \( F_{ev} \). It is important to exploit this roller-measured stiffness dependency on \( F_{ev} \) for CCC applications, and because IC employs variable \( F_{ev} \) with the objective of compacting soil more efficiently and uniformly (i.e., by using higher \( F_{ev} \) for areas with lower stiffness). If the amplitude dependence of roller-measured stiffness is not understood, it is very difficult to design the control logic for an IC roller, as the control parameter (roller-measured stiffness) would have unknown dependence on the quantity being controlled (\( F_{ev} \)). Further, an unknown or unclear understanding of the dependence of \( F_{ev} \) limits the usefulness of roller-measured stiffness during variable amplitude IC operation.

**Fig. 8.** Relationship between Ammann \( k_s \) and \( F_{ev} \) from roller passes on clayey sand test bed

![Graph](image-url)
Fig. 9 illustrates the relationship between in situ $M_T$, $M_S$, and Ammann $k_s$ from roller passes on clayey sand test bed. Rock are placed and compacted atop the sandy silt. $E_{vib}$ plateaus around 180 MPa at a total crushed rock thickness of 1.2–1.5 m above the silt, suggesting a 4:1–5:1 stiffness ratio (crushed rock to silt). At this point, roller measured stiffness is no longer influenced by the underlying sandy silt. Light weight deflectometer testing (Mooney and Miller 2008) was used to confirm the stiffness contrast in the materials. For the more typical base/subbase over subgrade thickness observed in practice and explored here (e.g., 0.3–0.6 m), roller-measured stiffness is clearly a composite measure influenced by both the crushed rock and silt.

In layered systems, roller-measured stiffness does not always parallel in situ stress-strain-modulus behavior. Fig. 11 illustrates the relationships observed between roller-measured stiffness (Bomag $E_{vib}$), in situ $M_T$ and $M_S$, and $F_{ev}$ collected during testing on the crushed rock over silt test bed. Recall in this test bed, stress-strain sensors were placed in the sandy silt and in the first layer of crushed rock (see inset in Fig 11). Multiple depth data were collected by adding layers of crushed rock, and thus changing the depth of the sensors from the surface. Moduli data from the silt are not available due to malfunction of the strain sensor. Within the data from each layer (layers 1, 2, 3 in Figs. 10 and 11 insets), $E_{vib}$ values increased with increasing $F_{ev}$ [Fig. 11(a)], yet $M_T$ and $M_S$ in the crushed rock both decrease with increasing $F_{ev}$ [Figs. 11(b and c)]. Focusing on $M_T$ behavior because it more closely parallels the definition of the $E_{vib}$ stiffness measurement (Figs. 2 and 6), the decrease in $M_T$ with increasing $F_{ev}$ at $z=0.39$ and 0.63 m is consistent with shear stress dependent, strain softening modulus behavior. Similar to clayey sand behavior, the $M_T$ versus $F_{ev}$ sensitivity increased with depth, and was reasonably insensitive at $z=0.23$ m.

A close inspection of the in situ stress data indicate that the layered structure is the reason for the increase in $E_{vib}$ despite the decrease in crushed rock $M_T$. As $F_{ev}$ increases, the ratio of the measured $\sigma_f$ in the crushed rock to $\sigma_f$ in the sandy silt increases, indicating that the much stiffer crushed rock material takes on a greater portion of the load (see Table 3). It is reasonable to assume that the ratio of strain in the crushed rock to silt follows suit (recall strain data in silt are not available). As $F_{ev}$ increases and the stiffer crushed rock takes more of the load, the roller-
measured stiffness is more reflective of the crushed rock and would therefore increase with increasing $F_{ev}$. It is believed that this effect is greater than the stress softening effect observed in Figs. 11(b and c), and results in the increase in roller-measured stiffness with increasing $F_{ev}$ observed in Fig. 11.

The relationship between composite stiffness (i.e., roller-measured stiffness) and the soil involves the interplay of a number of factors. First, as has been shown here, the stress levels induced by a vibrating roller—even at low amplitudes—result in nonlinear (stress-dependent) soil modulus behavior. The nature and degree of stress dependence (hardening or softening) is soil dependent (Ishihara 1996; Andrei et al. 2004). While the soils instrumented here exhibited softening behavior, it is conceivable that the mean normal effective stress increases induced by the roller could lead to hardening behavior in some soils. To this end, the stress-dependent modulus of a material may contribute to a composite stiffness increase or a decrease with increasing $F_{ev}$.

A second important factor is the increasing drum/soil contact area with increasing $F_{ev}$. The contact area plays a significant role in stress-strain profiles and in composite (total) response, in both homogeneous and layered systems (Burmister 1943). Isotropic, linear elastic theory applied to homogeneous and layered half space problems have shown that an increase in contact area results in increased composite stiffness (Timoshenko and Goodier 1951; Burmister 1943). The interplay of these two factors coupled with the stiffness ratio of the two layered materials and likely other factors (e.g., influence of roller/soil dynamic interaction) yields the $F_{ev}$ dependent roller stiffness results presented here. A rigorous analysis of dynamic drum/soil interaction accounting for layered, nonhomogeneous media with stress and rate-dependent moduli is a logical next step in exploring the complex interaction of the various factors. Such an analysis is beyond the scope of this paper.

### Table 3. Ratio of $\sigma_i$ in Base to $\sigma_i$ in Subgrade

<table>
<thead>
<tr>
<th>$F_{ev}$ (kN)</th>
<th>$\sigma_i, \text{base}/\sigma_i, \text{subgrade}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>87</td>
<td>1.60</td>
</tr>
<tr>
<td>175</td>
<td>1.77</td>
</tr>
<tr>
<td>250</td>
<td>1.81</td>
</tr>
</tbody>
</table>

### Conclusions

An experimental investigation was conducted to characterize in situ stress-strain behavior during vibratory roller loading on compacted soil, and to relate in situ behavior to roller-measured soil stiffness. One vertically homogeneous clayey sand test bed and two layered granular base over subgrade test beds were carefully constructed with embedded stress and strain sensors for the testing program. Multiple passes were performed with two IC rollers using various excitation force amplitudes and frequencies.

Total normal stress and strain measurements at multiple soil depths revealed complex triaxial behavior during vibratory roller loading. First, the stress state is variable along the 2.1 m length of the drum, with plane strain conditions observed under the drum’s center. Second, small values of $\sigma_i$ relative to $\sigma_y$ and $\sigma_z$, led to significant induced deviatoric stresses. Finally, the forward motion of the roller creates a bow wave effect that induces asymmetric conditions with vertical extension and longitudinal compression in front of the drum.

Measured cyclic strain amplitudes were on the order of $10^{-3}$ but were 15–25% of those measured during static roller passes. This is an important finding because roller-measured stiffness is derived from the cyclic displacement of the drum, which in turn is a reflection of the cyclic response of the soil. The lower cyclic strain levels (relative to static-induced strains) are attributed to viscoelasticity and curved drum/soil interaction. In the granular soils, the vibration-induced in situ stress and strain oscillated about the static-induced response. In the clayey sand test bed, vibration-induced strain was greater than static-induced response, and suggests the occurrence of modulus degradation, possibly due to vibration-induced pore pressures.

An examination of nonconstitutive vertical deformation moduli $M_T$ and $M_S$ extracted from in situ stress-strain response revealed vibratory roller loading—even under low $F_{ev}$ levels—induces nonlinear modulus behavior. In all test beds, $M_T$ and $M_S$ decreased with increasing $F_{ev}$ and increased with depth. These results indicate that roller measured stiffness is reflecting significant nonlinear modulus with depth. If roller-measured stiffness values are to be used to analyze pavement performance, the differences between roller-induced and traffic-induced stress conditions and their effect on modulus must be considered. Further, IC involves the continuous variation of $F_{ev}$ and the use of roller-
measured stiffness to guide the variation of $F_{ev}$. The dependence of roller-measured stiffness on the stress-strain state, and thus $F_{ev}$, creates a challenge for IC, where the measurement parameter should be independent of the control variable $F_{ev}$.

Measured soil stress and strain behavior helps to explain the $F_{ev}$ dependent roller measured stiffness observed throughout testing. On the vertically homogeneous clayey sand, roller-measured stiffness decreased with increasing $F_{ev}$. This behavior is attributed to the stress-dependent modulus reduction observed in situ. On the crushed rock over silt layered test bed, roller-measured stiffness increased with increasing $F_{ev}$ despite the mild stress-dependent modulus reduction observed in the crushed rock. Here, the ratio of $\sigma_i$ in the crushed rock to $\sigma_i$ in the underlying silt increased with $F_{ev}$ indicating that the crushed rock takes on a greater portion of the load. Because the crushed rock is 4–5 times stiffer than the underlying silt, the composite roller-measured stiffness increases accordingly.

Roller-measured stiffness and its dependence on $F_{ev}$ is influenced by the stress-dependent modulus function of each soil, the varying drum/soil contact area, and by layer characteristics (modulus ratio, thickness) when layering is present. The interplay between these and other factors (e.g., dynamics, rate dependent response) will dictate the nature of roller-measured stiffness dependence on $F_{ev}$.

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Notation

The following symbols are used in this paper:

- $a$ = one-half of drum/soil contact width (m);
- $E_{ vib}$ = Bomag roller-measured soil stiffness (kN/m²);
- $F_{ ev}$ = amplitude of vertical component of eccentric excitation force (kN);
- $F_i$ = contact force between drum and soil (kN/m);
- $F_{i(v)}$ = vertical component of eccentric excitation force (kN);
- $f$ = excitation frequency (Hz);
- $g$ = acceleration due to gravity (9.81 m/s²);
- $k_i$ = Ammann roller-measured soil stiffness (kN/m);
- $M_S$ = secant modulus extracted $\sigma_{si} - \varepsilon_{i}$, response (nonconstitutive);
- $M_T$ = tangent modulus extracted $\sigma_{ci} - \varepsilon_{c}$, response (nonconstitutive);
- $m_f, m_i$ = mass of frame and drum, respectively (kg);
- $m_{i(e)}$ = eccentric mass moment (kg m²);
- $m_{j(e)}$ = acceleration of frame and drum, respectively (m/s²);
- $P$ = drum/soil force per unit drum length (kN/m);
- $P_o$ = maximum normal drum/soil contact stress (kN/m²);
- $z_f, z_d$ = displacement of frame and drum, respectively (m);
- $\varepsilon_i$ = normal strain in i direction, $i=x,y,z$;
- $\varepsilon_{ic}$ = normal cyclic strain in i direction, $i=x,y,z$;
- $\psi = \text{Poisson's ratio};$
- $\sigma_i = \text{total normal stress in i direction, } i=x,y,z$
- $\sigma_{ic} = \text{total normal cyclic stress in i direction, } i=x,y,z$
- $\Omega = \text{excitation frequency (rad/s)}$.

References


